Homework 1 (Graded)

Published | Edit | ⋮

Due Friday, 6/28

ASV Exercises 1.43, 1.48, 1.50, 1.63 (9 points)

S. Grant Exercises:

S1.1: Prove via counting that $\sum_{(x_1,\ldots,\, x_k)\ s.t.\ x_1+\cdots+\, x_k=n,\ x_i\, \in\, \{0,1,\ldots,n\}} \binom{n}{x_1,\ldots,\, x_k} = k^n$. In words, "prove that the sum, over all positive integer k-tuples that add up to n, of n multi-choose x-one through x-k, is equal to k to the n." Note that $\binom{n}{x_1,\ldots,\, x_k} = \frac{n!}{x_1!\cdots x_k!}$ is the number of ways of grouping *n* unique items into *k* groups of sizes $x_1, \ldots, x_k$ so that if *k* = 2 it is simply a binomial coefficient like we saw in class. (2 points)

S1.2: A Keebler Elf faces a 2-meter by 2-meter wall **from one meter away**, at the center of which is a $\frac{1}{\sqrt{3}}$-meter *radius* circular target. The elf throws a dart once at this target from a height of 1 meter (thus **the dart begins its flight exactly one meter outward from the center of the wall and target**). Hitting the target wins better working conditions for all the Keebler Elves, but missing the wall means the elf gets fired. However, tired from a 14-hour day baking cookies with no lunch break, the elf's aim is off. They want to throw straight at the center of the target, but the angle of their aim (away from the straight line from the dart to the center of the wall/target) is chosen uniformly at random from $\left[-\frac{\pi}{4},\ \frac{\pi}{4}\right]$vertically and, independently,$\left[-\frac{\pi}{4},\ \frac{\pi}{4}\right]$horizontally.

- What is the probability the elf gets fired? (1 point)
- What is the probability the elf hits the upper half of the wall OR the left half of the wall? (1 point)
- What is the probability the elf hits the upper half of the wall AND the left half of the wall? (1 point)
- What is the probability the elf hits the smallest square containing the circular target? (.5 point)
- HARD: you will not be able to calculate the exact probability of the elf hitting the circular target until MATH/STAT 395. However, we can say something weaker now. Prove that the probability the elves get improved working conditions is less than 1/2 but greater than 1/5. Hint: use Kolmogorov axiom iii--the previous part gives you one of the bounds, and you can do another trick with a square to get the other bound. (.5 point)

Points	15
Submitting	on paper

Due	For	Available from	Until